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Origin of Internal Symmetries*

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Internal symmetries such as isotopic spin are not necessarily arbitrary constraints to be imposed at the beginning of a calculation. The bootstrap requirement that all particles be determined as composite states of one another leads naturally to symmetric solutions for masses and coupling constants.

I. INTRODUCTION

PHYSICAL systems are characterized by quantum numbers of energy, momentum, spin, and parity, whose origin is well understood; they arise from assumed symmetries of space-time. Some systems are also characterized by the internal quantum numbers of isotopic spin, hypercharge, and, more generally and less exactly, unitary spin, whose origins are less clear. We believe that these quantum numbers and the associated symmetries are already implied by the bootstrap mechanism of S-matrix theory.[1] There is no need for additional principles either inside or outside quantum theory to explain them.

The fundamental point is this: The internal symmetries can be expressed as equalities among certain masses and among certain couplings. But the values of these masses and couplings are not inserted into the theory as initial data; rather they emerge from a self-consistent calculation. Let us anticipate that in a fully self-consistent universe there is room for a multiplicity of particles of the same species, that is, of the same spin and parity. The formal principles which instruct us how to determine the masses and couplings of particles of like species possess a symmetry with regard to these particles. That such symmetries lead to equality among the masses and interactions of particles of like species need not be regarded as a freak accident, but may well be the preferred possibility.

In the present paper this quite general notion will be explored only with regard to pion-nucleon interactions and isotopic symmetry.

II. BOOTSTRAPS FOR PION-NUCLEON SYSTEMS

Imagine that a search is made for the simplest universe which includes spin $\frac{1}{2}$ particles, but that the attempt to construct such a universe self-consistently, in the bootstrap sense, with a single spin $\frac{1}{2}$ particle fails. Next, one might consider two such particles; call them nucleons and label them p and n. Space-time symmetries require that we consider the antiparticles $\bar{n}$ and $\bar{p}$ at the same time.

Now we ask what additional stable particles must be considered so that the family of particles as a whole is closed, i.e., so that each particle is, in fact, a bound state in one of the channels defined by the family. One can define, and perhaps even solve, a theory in which no additional particles are present, and in which each nucleon is a bound state of two nucleons and an antinucleon. One alternative is to suppose that stable states of nucleon-antinucleon systems, i.e., π mesons, also exist. Proceeding to examine all hypothetical systems in order of their (apparent) simplicity, we may first suppose that there is only one such meson, π^0, which couples to both $\bar{p}p$ and $\bar{n}n$. Then p is a bound state of $p\pi^0$ (and of all channels coupled to $p\pi^0$ as well), n is a bound state of $n\pi^0$, and π^0 is a bound state of $\bar{p}p$ and $\bar{n}n$. Thus π^0 must be its own antiparticle.

Figure 1 displays the lowest order-force diagrams (diagrams with a "left-hand" cut) leading to dynamical equations for the p, n, and π^0 bootstraps in this model. Figure 1(d) is included in the π^0-strap diagrams to illustrate that $\bar{p}p$ and $\bar{n}n$ are indeed coupled.

Still another possibility is that p and n are directly coupled through "charged" poins and that π^0 is not present. Thus we have a π^+ as a bound state of $\bar{n}p$, and necessarily, we have its antiparticle π^-, with the same mass, coupled to $\bar{p}n$. Figure 2 shows the lowest order diagrams for the various straps.

To test these schemes fully would require calculations beyond our powers; the most pleasing result would be that such self-supporting mechanisms cannot exist because the resultant forces are either repulsive or too weakly attractive, and that this is the reason that they do not occur in nature.

Our search then brings us to the observed case of two nucleons and three pions. The pions are anticipated to be of like species and, in fact, bound states of a single spin-parity nucleon-antinucleon partial wave (the $^1S_0$

* This work was supported in part by the Air Force Office of Scientific Research.

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[1] The general philosophy of bootstraps has been described in various papers, for example, F. Zachariasen and C. Zemach, Phys. Rev. 128, 849 (1962).

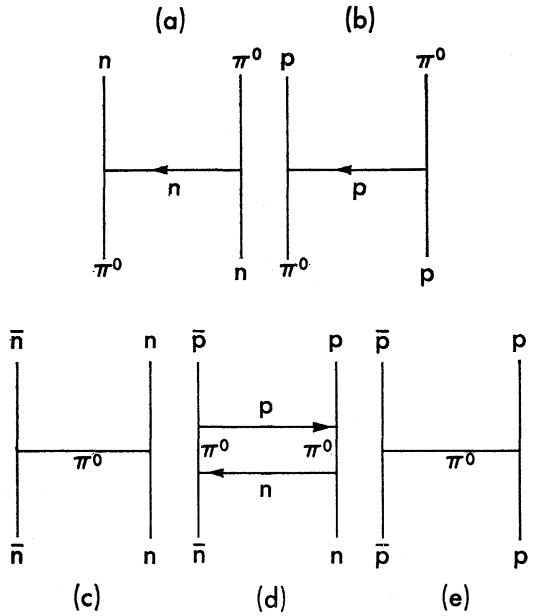

FIG. 1. Lowest order-force diagrams for a system containing only p, n, and π^0.

wave). The couplings are specified by the interaction scheme

$$g_c\bar{p}n\pi^+ + g_c\bar{n}p\pi^- + g_0\bar{p}p\pi^0 + h_0\bar{n}n\pi^0, \tag{2.1}$$

which may be regarded as an interaction Lagrangian in a field theory, but which the "pure" S-matrix theorist will prefer to interpret with statements about singularities of physical scattering amplitudes. We suppose, in any case, that (2.1) defines a meaningful dynamical structure whose self-consistency can be tested by calculation. The dependence of (2.1) on spin matrices is not indicated explicitly. The coupling constants will be real by time reversal invariance. The nucleon masses, M_n and M_p, are not yet assumed equal, nor are the various coupling constants. However, CPT requires π^+ and π^- to have the same mass μ_c, and the same coupling g_c to $\bar{p}n$ or $\bar{n}p$, respectively.

Consider now the dynamics of the p strap, by which we mean the dynamics of reactions with quantum numbers of p. Some of the forces are illustrated in Figs. 3(a) and 3(b). By looking at poles and residues of the amplitudes for

$$\pi^0+p \leftrightarrow \pi^0+p, \tag{2.2a}$$

$$\pi^0+p \leftrightarrow \pi^++n, \tag{2.2b}$$

$$\pi^++n \leftrightarrow \pi^++n, \tag{2.2c}$$

and imposing self-consistency, one obtains expressions for M_p, g_0^2, g_0g_c, g_c^2 having the form

$$M_p = F_a(M_p,M_n,\mu_c,\mu_0,g_c,g_0,h_0), \tag{2.3a}$$

$$g_0^2 = F_b(M_p,M_n,\mu_c,\mu_0,g_c,g_0,h_0), \tag{2.3b}$$

$$g_0g_c = F_c(M_p,M_n,\mu_c,\mu_0,g_c,g_0,h_0), \tag{2.3c}$$

$$g_c^2 = F_d(M_p,M_n,\mu_c,\mu_0,g_c,g_0,h_0). \tag{2.3d}$$

Only the sign of g_0 relative to g_c is relevant; let g_c be positive so that the sign of g_0 is also fixed. Of course, $F_c^2=F_bF_d$ will hold identically and does not constitute and additional relation. A similar procedure with the n strap yields expressions for M_n, etc. But the n problem is transformed into the p problem by the following interchanges of notation in (2.1):

$$p \leftrightarrow n \quad (\text{including } M_n \leftrightarrow M_p),$$
$$g_0 \leftrightarrow h_0,$$
$$\pi^+ \leftrightarrow \pi^-.$$

Then we must have

$$M_n = F_a(M_n,M_p,\mu_c,\mu_0,g_c,h_0,g_0), \tag{2.4a}$$

$$h_0^2 = F_b(M_n,M_p,\mu_c,\mu_0,g_c,h_0,g_0), \tag{2.4b}$$

$$h_0g_c = F_c(M_n,M_p,\mu_c,\mu_0,g_c,h_0,g_0), \tag{2.4c}$$

$$g_c^2 = F_d(M_n,M_p,\mu_c,\mu_0,g_c,h_0,g_0). \tag{2.4d}$$

From Eqs. (2.3), eliminate g_0, h_0 to obtain an equation of the type

$$M_p = \mathfrak{F}(M_p,M_n;\mu_c,\mu_0,g_c). \tag{2.5a}$$

The same treatment of Eqs. (2.4) yields

$$M_n = \mathfrak{F}(M_n,M_p;\mu_c,\mu_0,g_c) \tag{2.5b}$$

with the same function $\mathfrak{F}$. By considering simple models, of the N/D type, for example, one may convince oneself that (2.5a), (2.5b) are meaningful equations, not identical, and for fixed values of μ_c, μ_0, and g_c have a discrete number of solutions, if any, for M_n and M_p. An example of such a model is discussed in detail in the next section. Equation (2.5a) represents a curve in the M_p, M_n plane and (2.5b) represents the reflection of that curve through the line $M_p=M_n$. Where these curves

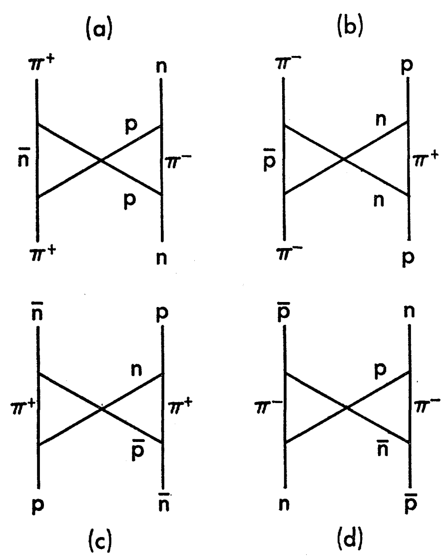

FIG. 2. Lowest order-force diagrams for a system containing only p, n, π^+ and π^-.

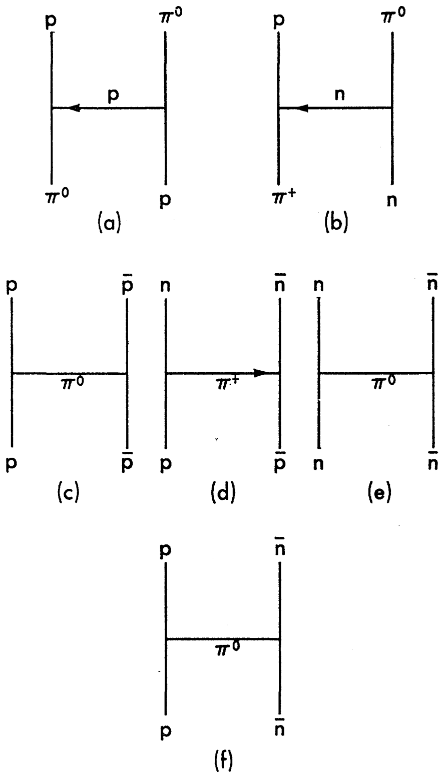

FIG. 3. Some lowest order-force diagrams for the 3-pion, 2-nucleon system.

intersect there is a possible pair of values M_p, M_n. If either curve goes through the line $M_p=M_n$, the two will intersect there yielding a solution with $M_p=M_n$. For a large class of functions, the only possible solution to (2.5a) and (2.5b) is $M_p=M_n$. In this way equality of the nucleon masses may arise *naturally* from a dynamical calculation. We have not shown that M_n and M_p *must* be equal, but that this fact of nature can be the plausible outcome of the calculation rather than a freak accident.

A similar manipulation of Eqs. (2.3) and Eqs. (2.4) demonstrates the plausibility, though not the necessity, that $g_0^2=h_0^2$ or $g_0=\pm h_0$.

The G transformation is usually defined on nucleons in such a way that nucleon states are replaced by their antiparticle states with charges and phases adjusted as follows:

$$\begin{aligned} p &\to \bar{n}, & \bar{p} &\to n, \\ n &\to -\bar{p}, & \bar{n} &\to -p. \end{aligned} \tag{2.6}$$

Then the following states, in the ${}^1S_0$ partial wave, have negative G parity:

$$\bar{n}p,\quad (\bar{p}p-\bar{n}n)/\sqrt{2},\quad \bar{p}n, \tag{2.7a}$$

while

$$(\bar{p}p+\bar{n}n)/\sqrt{2} \tag{2.7b}$$

has positive G parity. Thus π^+ and π^- have negative G parity, as does π^0 if $g_0=-h_0$. G parity of the Lagrangian is conserved, of course, regardless of whether $g_0=+h_0$ or $g_0=-h_0$. We shall assume hereafter that $M_n=M_p$ and $g_0=-h_0$ do emerge from the bootstrap calculations as has been made plausible. There may well be another meson, the η, coupled to $(\bar{p}p+\bar{n}n)/\sqrt{2}$. Its presence would affect the dynamics, but not the symmetry properties of the odd G-parity states, so that it is irrelevant to the subsequent arguments.

The result so far constitutes charge symmetry; to establish charge independence, we consider the pion straps. We first define new pion states of definite mass μ_c by

$$\pi^1=(\pi^++\pi^-)/\sqrt{2},$$
$$\pi^2=i(\pi^+-\pi^-)/\sqrt{2},$$

and then rewrite (2.1) in the notation of isotopic spin:

$$g_1\bar{N}\tau_1N\pi^1+g_2\bar{N}\tau_2N\pi^2+g_0\bar{N}\tau_3N\pi^0, \tag{2.8}$$

where

$$g_1=g_2=g_c/\sqrt{2}.$$

N stands for the isospinor with components p, n. We write μ_1, μ_2 for the masses of the π^1, π^2 although, of course, $\mu_1=\mu_2=\mu_c$.

From the poles and residues of the amplitudes for

$$\bar{N}+N\leftrightarrow\bar{N}+N,$$

we obtain, as before, expressions for μ_1, g_1^2 of the type (we have set $M_p=M_n=M$):

$$\mu_1=f_a(\mu_1,\mu_2,\mu_0,g_1,g_2,g_0,M), \tag{2.9a}$$

$$g_1^2=f_b(\mu_1,\mu_2,\mu_0,g_1,g_2,g_0,M). \tag{2.9b}$$

Next, change the *description* of the nucleons, defining a new isospinor N' by the following two-step process:

(a) an isotopic rotation of the nucleons such that $\bar{N}\tau_1N\to-\bar{N}'\tau_3N'$, $\bar{N}\tau_2N\to-\bar{N}'\tau_2N'$, and $\bar{N}\tau_3N'\to-\bar{N}'\tau_1N'$. In three-dimensional isotopic space, this is a rotation by 180° about the line $x+z=0$, $y=0$. And then

(b) a G transformation of the nucleons as defined in (2.6).

Then (2.8) takes the form

$$g_1\bar{N}'\tau_3N'\pi^1+g_2\bar{N}'\tau_2N'\pi^2+g_0\bar{N}'\tau_1N'\pi^0. \tag{2.10}$$

Since (2.10) is *formally* identical to (2.8) under the substitutions

$$\begin{aligned} &\pi^1\leftrightarrow\pi^0 \quad (\text{with } \mu_1\leftrightarrow\mu_0), \\ &g_1\leftrightarrow g_0, \end{aligned} \tag{2.11}$$

we have, also,

$$\mu_0=f_a(\mu_0,\mu_2,\mu_1,g_0,g_2,g_1,M), \tag{2.12a}$$

$$g_0^2=f_b(\mu_0,\mu_2,\mu_1,g_0,g_2,g_1,M). \tag{2.12b}$$

The elimination of g_1^2 and g_0^2 from Eqs. (2.9) and Eqs. (2.12) yields equations of the type

$$\mu_0=\tilde{f}(\mu_0,\mu_1;\mu_2,g_2,M), \tag{2.13a}$$

$$\mu_1=\tilde{f}(\mu_1,\mu_0,;\mu_2,g_2,M). \tag{2.13b}$$

From these, as before, we infer the plausibility of $\mu_0=\mu_1$. The same argument suggests that $g_0^2=g_1^2=\frac{1}{2}g_c^2$ is a natural consequence of the dynamics. Then, if g_0 and g_c have the same sign, the interaction coincides with the customary isotopic-spin invariant pion-nucleon interaction. If g_0 and g_c have opposite signs, this is also true except we must identify the n states with protons and the p states with neutrons.

Notice that in order to infer the likelihood of isotopic invariance, it was not necessary to consider all isotopic rotations or even infinitesimal ones, but only two permutation operations. This will not be surprising to those who remember that the original inference of isotopic spin from experiment rested solely on the equivalence of pp and nn forces and the equivalence of pp and pn forces.[2]

The possibility of bootstrap theories embodying isospin symmetry is, of course, well known. The primary aim of the present discussion has been to show how dynamics may rule out other alternatives.

III. A MODEL

We shall now illustrate how the abstract arguments of the previous section, involving the fictitious particles π^1 and π^2, take form in a simple model calculation.

In the model, we shall deal only with the second half of the problem. Suppose that the first results (charge symmetry) have already been proved: namely, $M_n=M_p$ and $g_0=-h_0$. (We assume the case $g_0=+h_0$ has been disposed of.) It is important to emphasize that $M_n=M_p$ and $g_0=-h_0$ are *not* to be considered as postulates, but are expected to follow from an explicit dynamical calculation of the nucleon strap. In fact, we could have constructed a model analogous to the one described below which predicts an explicit form for Eq. (2.5) and from which, independent of any assumptions on the pion masses, we could deduce $M_n=M_p$ and $g_0=-h_0$. We calculate the $\bar{N}N$ partial wave with the space-time quantum numbers of the pion by the N/D method, assuming that the lowest order exchange diagrams are sufficient to define the forces (left-hand cuts) which bind $\bar{n}p$ to form π^+ [Fig. 3(f)] and which bind $\bar{p}p$ and $\bar{n}n$ to form π^0 [Figs. 3(c), 3(d), 3(e)]. This is, of course, a very poor approximation dynamically, but is still expected to contain the symmetries under discussion. Finally, we approximate the left-hand cut by a single pole. This pole must lie to the left of the bound-state poles we seek.

For a given diagram, we shall place the pole at $s=s_t-4\mu^2$, where μ is the exchanged pion mass, and $s_t=4M^2$ is the threshold energy squared.

Then the Born amplitude for the process

$$p+\bar{n}\leftrightarrow p+\bar{n} \tag{3.1}$$

is given the form

$$t_B^+(s)=g_0^2A/(s-s_n)\,, \tag{3.2}$$

where s is the square of the center-of-mass frame energy, $s_n=s_t-4\mu_0^2$ is the pole position for neutral pion exchange, and A is some kinematical factor. A may, in principle, be a function of the exchanged mass, but we lose little generality by taking it to be a constant. A must be positive to produce an attractive force. The full amplitude is now represented by

$$t^+(s)=N(s)/D(s)\,. \tag{3.3}$$

We use the following unitarity condition

$$\mathrm{Im}t^+=(s-s_t)^{\frac{1}{2}}|t^+|^2 \quad \text{for} \quad s>s_t\,. \tag{3.4}$$

Then, in the usual way

$$N(s)=t_B^+(s)\,, \tag{3.5}$$

$$D(s)=1-\frac{s-s_n}{\pi}\int_{s_t}^{\infty}\frac{(s'-s_t)^{\frac{1}{2}}N(s')}{(s'-s)(s'-s_n)}ds'\,. \tag{3.6}$$

We define

$$z_n=(s_t-s_n)^{\frac{1}{2}}\,, \tag{3.7}$$

$$z=(s_t-s)^{\frac{1}{2}}\,, \tag{3.8}$$

$$z_+=(s_t-\mu_c^2)^{\frac{1}{2}}\,. \tag{3.9}$$

Substituting (3.5) into (3.6), we have[3]

$$D(s)=1-g_0^2A[(z+z_n)^{-1}-(2z_n)^{-1}]\,. \tag{3.10}$$

The output amplitude [annihilation graph, Fig. 4(d)] is

$$t_{\text{out}}^+=-g_c^2B/(s-\mu_c^2)\,, \tag{3.11}$$

where B is some other kinematical factor. The π^+ mass is determined by the condition

$$D(\mu_c^2)=0\,, \tag{3.12}$$

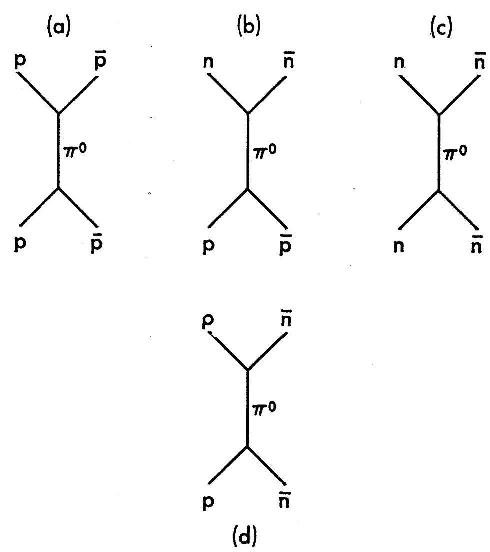

FIG. 4. Annihilation graphs in the $\bar{N}N$ straps.

[2] B. Cassen and E. U. Condon, Phys. Rev. **50**, 846 (1936).

[3] G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1961), p. 53.

and the $\bar{p}n\pi$, coupling by

$$g_c^2B=-N(\mu_c^2)/D'(\mu_c^2)\,, \tag{3.13}$$

where $D'(s)=dD(s)/ds$. The explicit solution is

$$z_+=z_n(g_0^2A-2z_n)/(g_0^2A+2z_n)\,, \tag{3.14a}$$

$$g_c^2B=g_0^2Az_+/z_n\,. \tag{3.14b}$$

In this way the charged pion parameters are determined from the input π^0 parameters. We can now add them to the input information to calculate output values for μ_0^2 and g_0^2. All four quantities will then be determined by a reciprocal bootstrap mechanism.

The π^0 strap is a two-channel problem. We label the $\bar{p}p$ and $\bar{n}n$ channels 1 and 2, respectively, and obtain, for the Born amplitude t_B^0, the matrix

$$t_B^0(s)=\begin{bmatrix} \dfrac{-g_0^2A}{s-s_n} & \dfrac{-g_c^2A}{s-s_c} \\[2ex] \dfrac{-g_c^2A}{s-s_c} & \dfrac{-g_0^2A}{s-s_n} \end{bmatrix}. \tag{3.15}$$

Here, $s_c=s_t-4\mu_c^2$ is the position of the pole due to charged pion exchange. Because we have assumed the nucleon masses already equal, t_B^0 and all matrices which are functions of t_B^0 can be diagonalized by an energy-independent matrix U:

$$U=U^{-1}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}, \tag{3.16}$$

reducing this problem to a single-channel one also. We have

$$T_B^0(s)=Ut_B^0(s)U^{-1}=\begin{bmatrix} \dfrac{-g_c^2A}{s-s_c}-\dfrac{g_0^2A}{s-s_n} & 0 \\[2ex] 0 & \dfrac{g_c^2A}{s-s_c}-\dfrac{g_0^2A}{s-s_n} \end{bmatrix}. \tag{3.17}$$

In the original representation the output matrix contains the π^0 pole in the form

$$t_{\rm out}^0(s)=\frac{-B}{s-\mu_0^2}\begin{pmatrix} g_0^2 & -g_0^2\\ -g_0^2 & g_0^2\end{pmatrix}. \tag{3.18}$$

In the new representation, this becomes

$$T_{\rm out}^0(s)=\frac{-B}{s-\mu_0^2}\begin{pmatrix} 0 & 0\\ 0 & 2g_0^2\end{pmatrix}. \tag{3.19}$$

Thus the π^0 is coupled only to the antisymmetric combination $\bar{n}n-\bar{p}p$. Since $A>0$, this is the only consistent result; T_B^0 is attractive in this channel only.

TABLE I. Self-consistent solutions to model bootstrap equations.

A	$(\mu_0/2M)^2$	$(\mu_c/2M)^2$	g_0^2	$\frac{1}{2}g_c^2$
3.0	0.36	0.36	4.0	4.0
	0.68	0.21	3.6	2.9
3.5	0.42	0.42	2.76	2.76
	0.60	0.35	2.80	2.57
8.0	0.80	0.80	0.75	0.75

Now we can calculate the amplitude in this channel by the same N/D method, again normalizing D to 1 at s_n. We write $T^0(s)=N(s)/D(s)$, and the solution is

$$N(s)=\frac{g_c^2AD(s_c)}{s-s_c}-\frac{g_0^2A}{s-s_n}\,, \tag{3.20}$$

$$D(s)=1-\frac{s-s_n}{\pi}\int_{s_t}^{\infty}\frac{(s'-s_t)^{\frac{1}{2}}N(s')ds'}{(s'-s)(s'-s_n)}\,. \tag{3.21}$$

Then

$$D(s)=1-g_c^2AD(s_c)[(z+z_c)^{-1}-(z_n-z_c)^{-1}] +g_0^2A[(z+z_n)^{-1}-(2z_n)^{-1}], \tag{3.22}$$

where

$$z_c=(s_t-s_c)^{\frac{1}{2}}. \tag{3.23}$$

By setting $s=s_c$ in (3.22), we obtain a linear equation to determine $D(s_c)$. The bootstrap conditions are

$$D(\mu_0^2)=0\,, \tag{3.24a}$$

$$2g_0^2B=N(\mu_0^2)/D'(\mu_0^2)\,. \tag{3.24b}$$

Thus, z_0 can be found explicitly by solving a quadratic algebraic equation, and then g_0^2 follows directly by inserting $\mu_0^2=s_t-z_0^2$ into (3.24b).

Equations (3.14) and (3.24) form four bootstrap equations for the four unknown quantities. If values of g_0^2 and μ_0 exist such that the solutions to Eqs. (3.14) have the properties

$$\mu_c=\mu_0\,, \tag{3.25}$$

$$g_c^2=2g_0^2\,, \tag{3.26}$$

one finds by direct substitution that Eqs. (3.24) are automatically satisfied, so that these values are, in fact, solutions to the whole problem. They are precisely the "natural" solutions which embody isotopic spin symmetry.

To carry out the calculation, we take input values μ_0, g_0, find μ_c, g_c from (3.14), and then calculate output values $\bar{\mu}_0$, $\bar{g}_0$ from (3.24). The self-consistency conditions,

$$\mu_0=\bar{\mu}_0(\mu_0,g_0)\,,$$
$$g_0=\bar{g}_0(\mu_0,g_0)\,,$$

define curves in the g_0, μ_0 plane. The intersections, if any, of the $\mu_0=\bar{\mu}_0$ and $g_0=\bar{g}_0$ curves give the self-consistent values of μ_0, g_0, and then μ_c, g_c are given by (3.14). For $B=1$ and the three choices $A=3.0$, $A=3.5$, $A=8$ (in units such that $s_t=1$), the curves are given in Figs. 5(a)–(c), and the self-consistent values in Table I.

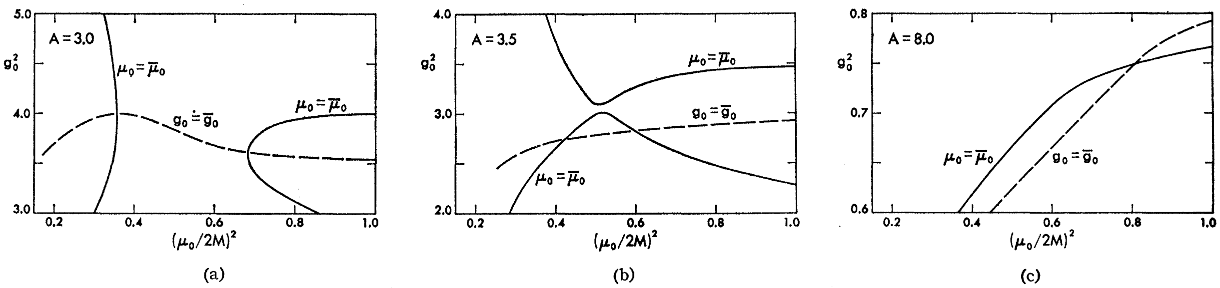

FIG. 5. The two self-consistency conditions for μ_0 and g_0^2.

The curves $\mu_0=\bar{\mu}_0$ may have more than one branch. In each case, the "natural" solution, embodying isotopic symmetry, is found, and in two of the cases, a second nonsymmetric solution is also found.

For very small values of A, no solution exists. There are apparently no values of A for which a nonsymmetric solution exists and a symmetric solution does not in this model, although there is no logical reason why this could not happen.

Other models embracing the features of this one may also be easily constructed. For example, given the existence of three pions, we infer from the bootstrap principle that pion pairs may form vector mesons, that is, ρ mesons. Since Bose statistics do not allow a vector state composed of two like pions, only three ρ mesons, built from the three pairs of unlike pions, can appear in such a theory. One can also build pions from $\pi\rho$ states, thus providing a crossed strap to enclose the system. From the π strap, one obtains symmetric equations for the charged and neutral pion masses which, in various calculational schemes (such as the one described, or, alternatively, the determinantal method), have equal pion masses as the only solution (without the necessity of assuming all ρ masses equal). Turning to the ρ strap and using the now known equality of the π masses, one can derive in the same model the equality of the ρ masses and the correct isospin relations among the coupling constants consistent with isospin one for both the pion and the ρ meson.

In conclusion, the bootstrap principle, which we would term "well understood," provides the key to many properties of nature which heretofore have not been understood, not even philosophically. Let us emphasize again the salient points. First, either the number of particle types, with their various spin-parity assignments, which exist in nature is uniquely determined, or at least the possibilities are greatly restricted. Second, equality of spins and parities among a set of particles is not merely consistent with equality of masses, but may well *guarantee* equality of masses in some cases. Thus, the remarkable fact that some physically distinguishable particles have equal masses (or at least equal apart from electromagnetic effects) is explainable in terms of principles that are already well understood. Third, certain ratios among coupling constants may also be guaranteed, and these, together with the mass equalities, define the relationships of an internal symmetry, such as isospin symmetry.[4] Fourth, several specific models support this view but also suggest that one cannot prove the necessity of isospin symmetry without rather detailed studies of the dynamics.

If this kind of argument is extended to, say, unitary spin, then the surprising thing in connection with strong interaction symmetries is not that they exist, but that they are broken. This may be a difficult *dynamical* problem.

[4] R. H. Capps [Phys. Rev. Letters **10**, 312 (1963)] has shown that, if mass equalities are assumed, the bootstrap principle implies the full SU_3 symmetry for interactions of pseudoscalar mesons and vector mesons.